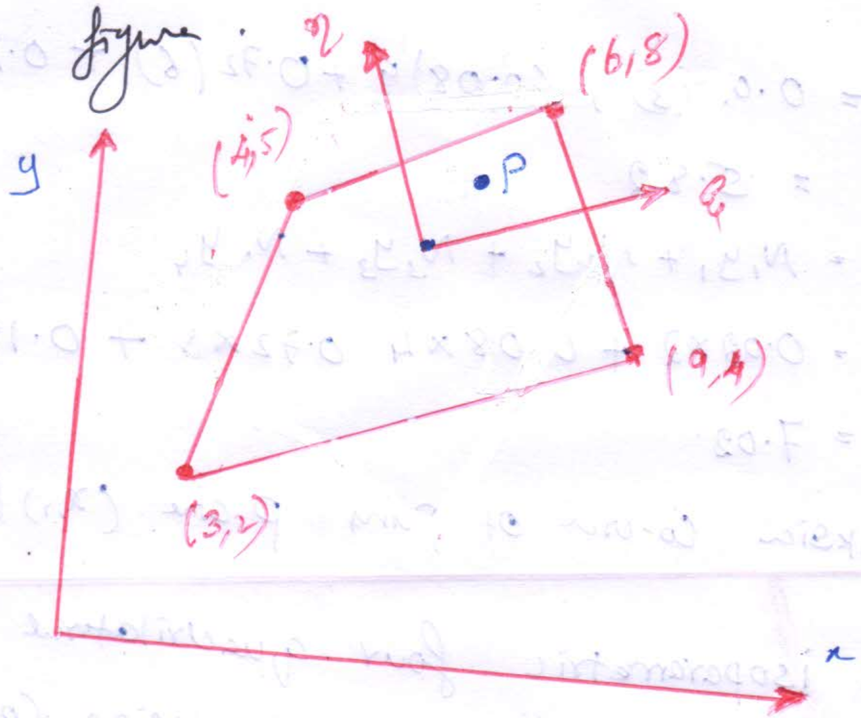
1) Evaluate the Cartesian Co-ordinate of the point P which has local Co-ordinates $\xi = 0.6$ & $\eta = 0.8$ as shown in figure.



Given: Co-ordinates of P, $\xi = 0.6$, $\eta = 0.8$

Cartesian Co-ordinates of point 1, 2, 3 & 4.

$x_1 = 3$, $x_3 = 6$, $y_1 = 2$ $y_3 = 8$

$x_2 = 9$, $x_4 = 4$ $y_2 = 4$ $y_4 = 5$

To find $P(x, y)$

We know that

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta) = \frac{1}{4}(1-0.6)(1-0.8) = 0.02$$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta) = \frac{1}{4}(1+0.6)(1-0.8) = 0.08$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta) = \frac{1}{4}(1+0.6)(1+0.8) = 0.72$$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta) = \frac{1}{4}(1-0.6)(1+0.8) = 0.18$$

$x = N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4$

$y = N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4$

$x = 0.02(3) + (0.08)9 + 0.72(6) + 0.18(4)$

$= 5.82$

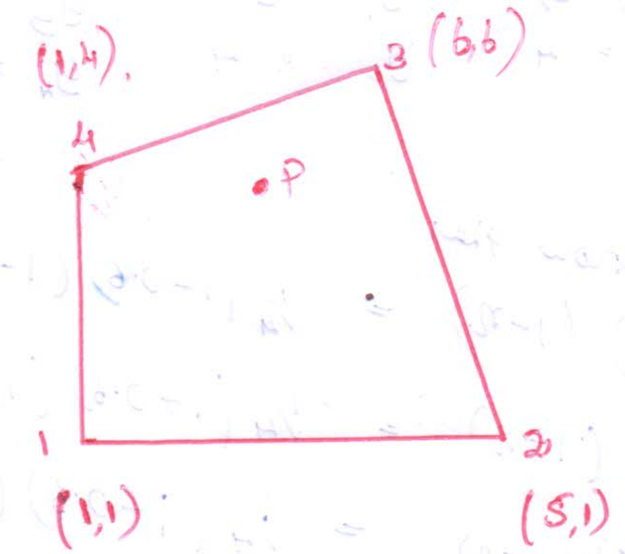
$y = N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4$

$= 0.02 \times 2 + 0.08 \times 4 \;\; 0.72 \times 8 + 0.18(5)$

$= 7.02$

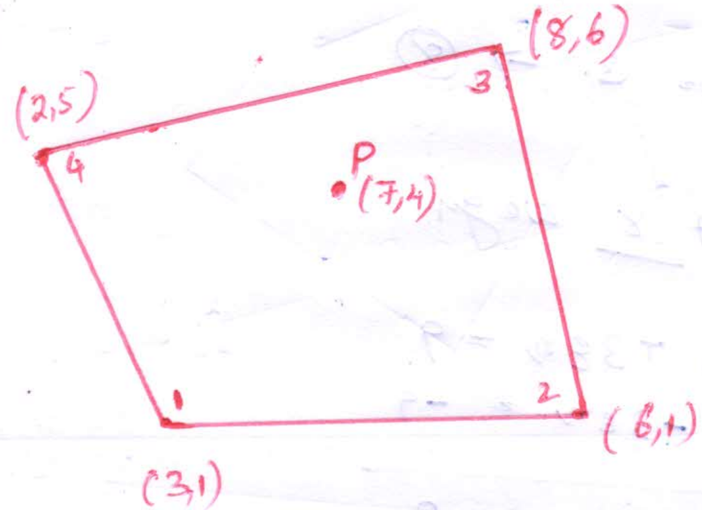
The Cartesian co-ordi of point P are (x, y) $(5.82, 7.02)$.

HW (2) For the isoparametric four quadrilateral element shown in fig. Determine the Cartesian co-ordinates of point P which has local co-ordinates $\varepsilon = 0.5, \eta = 0.5$



$(x, y) = (4.56, 4.38)$.

3. For the isoparametric quadrilateral element shown in figure determine the local co-ordinates of the point P which has Cartesian coord (7,4)



$x = 7 \quad y = 4$

$x_1 = 3 \quad y_1 = 1$

$x_2 = 6 \quad y_2 = 1$

$x_3 = 8 \quad y_3 = 6$

$x_4 = 2 \quad y_4 = 5$

We know

$$\left.\begin{array}{l} N_1 = \frac{1}{4}(1-\xi)(1-\eta), \quad N_2 = \frac{1}{4}(1+\xi)(1-\eta) \\ N_3 = \frac{1}{4}(1+\xi)(1+\eta) \quad N_4 = \frac{1}{4}(1-\xi)(1+\eta) \end{array}\right\} 1 \text{ to } 4$$

~~We k~~

$$x = N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4 \rightarrow (5)$$

$$y = N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4 \rightarrow (6)$$

sub (1) to (4) in (5)

$$7 = \frac{1}{4}\left[(1-\xi)(1-\eta)3 + (1+\xi)(1-\eta)6 + (1+\xi)(1+\eta)8 + (1-\xi)(1+\eta)2\right]$$

solvg

$$28 = 19 + \eta + 9\xi + 3\xi\eta$$

$$\boxed{\eta + 9\xi + 3\xi\eta = 9} \rightarrow (7)$$

sub 1 to 4 to 6. we get.

$$4 = \frac{1}{4}\left[(1-\varepsilon)(1-\eta)1 + (1+\varepsilon)(1-\eta)\times 1) + (1+\varepsilon)(1+\eta)6 + (1-\varepsilon)(1+\eta)\times 5\right]$$

$$9\eta + \varepsilon + \varepsilon\eta = 3. \rightarrow (8)$$

Resolv 7 + 8 we get

$$\eta + 9\varepsilon + 3\varepsilon\eta = 9.$$

$$8\times(-3)\ -27\eta - 3\varepsilon - 3\varepsilon\eta = -9$$

$$-26 + 6\varepsilon = 0.$$

$$-26\eta = -6\varepsilon$$

$$\varepsilon = 4.333\,\eta.$$

sub in (7)

$$\eta + 9\varepsilon + 3\varepsilon\eta = 9$$

$$\eta + 9(4.33\eta) + 3\cdot(4.33\eta)\eta = 9.$$

$$13\eta^2 + 40\eta = 9$$

$$\eta = \frac{-40 \pm \sqrt{40^2 - 4(13)(-9)}}{2(13)}$$

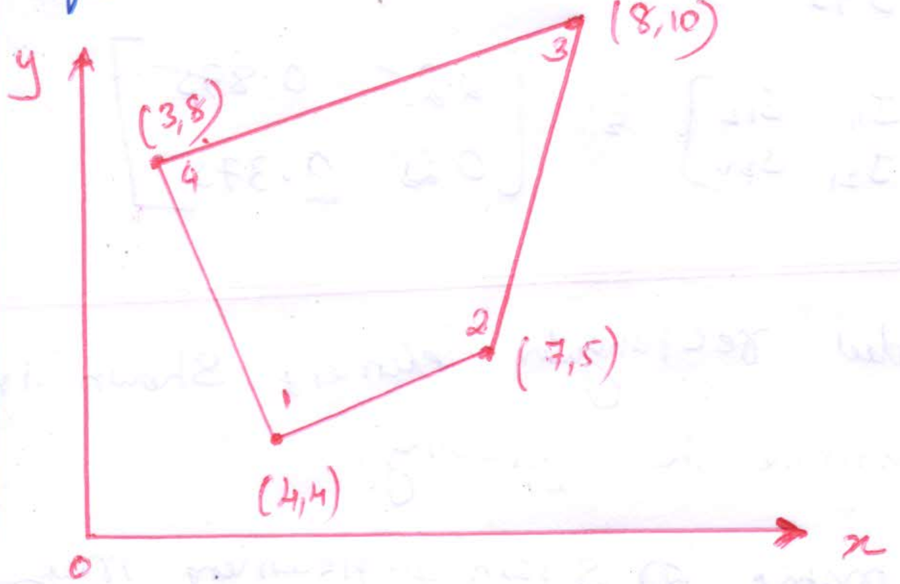
$$\eta = 0.210587$$

$$\varepsilon = 4.33 \times 0.210587$$

$$\varepsilon = 0.912545.$$

the local coor $(\eta, \varepsilon) = (0.2108, 0.913)$

(4) Evaluate [J] at $\varepsilon = \eta = \frac{1}{2}$ for the linear quadrilateral element shown in figure.



$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

$$J_{11} = \frac{1}{4}\left[-(1-\eta)x_1 + (1-\eta)x_2 + (1+\eta)x_3 - (1+\eta)x_4\right]$$

$$J_{12} = \frac{1}{4}\left[-(1-\eta)y_1 + (1-\eta)y_2 + (1+\eta)y_3) - (1+\eta)y_4\right]$$

$$J_{21} = \frac{1}{4}\left[-(1-\varepsilon)x_1 - (1+\varepsilon)x_2 + (1+\varepsilon)x_3 + (1-\varepsilon)x_4\right]$$

$$J_{22} = \frac{1}{4}\left[-(1-\varepsilon)y_1 - (1+\varepsilon)y_2 + (1+\varepsilon)y_3 + (1-\varepsilon)y_4\right]$$

$$J_{11} = \frac{1}{4}\left[-(1-0.5)4 + (1-0.5)7 + (1+0.5)8 - (1+0.5)3\right]$$

$$= 2.25$$

$$J_{12} = \frac{1}{4}\left[-[(1-0.5)]4 + (1-0.5)\times 5 + (1+0.5)10 - (1+0.5)8\right]$$

$$= 0.875$$

$$J_{21} = \frac{1}{4}\left[-[1-0.5)4 - [1+0.5]7 + [1+0.5]8 + [1-0.5]3\right]$$

$$= 0.25$$

$$J_{22} = \tfrac{1}{4}\left[-[1-0.5]4 - (1+0.5)5 + (1+0.5)10 + (1-0.5)8\right]$$

$$= 2.375$$

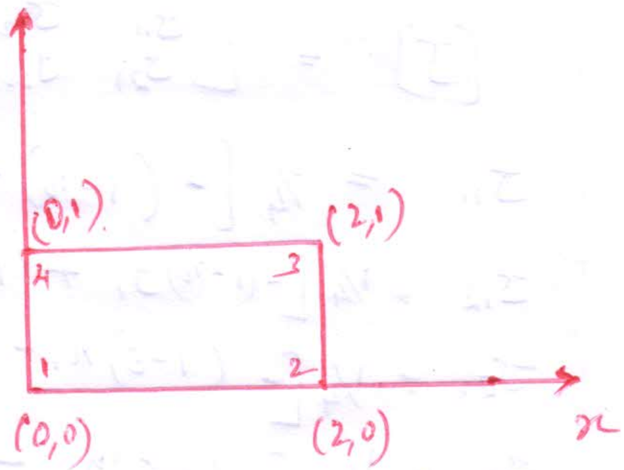
$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} 2.25 & 0.875 \\ 0.25 & 2.375 \end{bmatrix}$$

---

(5) A 4 noded rectangular element shown in figure. Determine the following.

(i) Jacobian matrix (2) Strain Displacement matrix

(3) Element Stresses.



Take $E = 2\times10^5\ N/mm^2$

$\mu = 0.25$

$u = [0, 0, 0.003, 0.004, 0.006, 0.004, 0, 0]^T$

$\varepsilon = 0,\ \eta = 0.$

Assume plane stress condition.

Soln:

$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

$$J_{11} = \tfrac{1}{4}\left[-(1-\eta)x_1 + (1-\eta)x_2 + (1+\eta)x_3 - (1+\eta)x_4\right]$$

$$J_{12} = \tfrac{1}{4}\left[-(1-\eta)y_1 + (1-\eta)y_2 + (1+\eta)y_3 - (1+\eta)y_4\right]$$

$$J_{13} = \tfrac{1}{4}\left[-(1-\varepsilon)x_1 - (1+\varepsilon)x_2 + (1+\varepsilon)x_3 + (1-\varepsilon)x_4\right]$$

$$J_{14} = \tfrac{1}{4}\left[-(1-\varepsilon)y_1 - (1+\varepsilon)y_2 + (1+\varepsilon)y_3 + (1-\varepsilon)y_4\right]$$

Solu

$J_{11} = 1, \quad J_{12} = 0, \quad J_{21} = 0, \quad J_{22} = 0.5$

$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$|J| = 0.5$$

$$[B] = \frac{1}{|J|} \times \begin{bmatrix} J_{22} & -J_{12} & 0 & 0 \\ 0 & 0 & -J_{21} & J_{11} \\ 0 & 0 & J_{22} & -J_{12} \\ -J_{21} & J_{11} & & \end{bmatrix} \times \frac{1}{4}$$

$$\begin{bmatrix} -(1-\eta) & 0 & -(1-\eta) & 0 & (1+\eta) & 0 & -(1+\eta) & 0 \\ -(1-\varepsilon) & 0 & -(1+\varepsilon) & 0 & (1+\varepsilon) & 0 & (1-\varepsilon) & 0 \\ 0 & -(1-\eta) & 0 & (1-\eta) & 0 & (1+\eta) & 0 & -(1+\eta) \\ 0 & -(1-\varepsilon) & 0 & -(1+\varepsilon) & 0 & (1+\varepsilon) & 0 & (1-\varepsilon) \end{bmatrix}$$

$$[B] = \frac{1}{0.5} \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0.5 & 0 \end{bmatrix} \times \frac{1}{4} \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$= \frac{0.5}{0.5 \times 4} \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & -2 & 0 & 2 & 0 & 2 \\ -2 & -1 & -2 & 1 & 2 & 1 & 2 & -1 \end{bmatrix}$$

$$[B] = 0.25 \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & -2 & 0 & 2 & 0 & 2 \\ -2 & -1 & -2 & 1 & 2 & 1 & 2 & -1 \end{bmatrix}$$

We know element stress.

$$\sigma = [D][B]\{u\} \rightarrow (1)$$

For plane stress condition, Stress strain relation matrix.

$$[D] = \frac{E}{1-\mu^2}\begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}$$

$$= \frac{2\times10^5}{1-(0.25)^2}\begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1-0.25}{2} \end{bmatrix}$$

$$[D] = 53.333\times10^3\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

sub $[D]$, $[B]\{u\}$ in eqn ①

$$\{\sigma\} = 53.33\times10^3\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}\times 0.25\begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & -2 & 0 & 2 & 0 & 2 \\ -2 & -1 & -2 & 1 & 2 & 1 & 2 & 1 \end{bmatrix}\times\begin{Bmatrix} 0 \\ 0 \\ 0.003 \\ 0.004 \\ 0.006 \\ 0.004 \\ 0 \\ 0 \end{Bmatrix}$$

$$\{\sigma\} = 53.33\times10^3\times0.25\begin{bmatrix} -4 & -2 & 4 & -2 & 4 & 2 & -4 & 2 \\ -1 & -8 & 1 & -8 & 1 & 8 & -1 & 8 \\ -3 & -1.5 & -3 & 1.5 & 3 & 1.5 & 3 & -1.5 \end{bmatrix}\times\begin{Bmatrix} 0 \\ 0 \\ 0.003 \\ 0.004 \\ 0.006 \\ 0.004 \\ 0 \\ 0 \end{Bmatrix}$$

$$\sigma = 13.333\times10^3\begin{Bmatrix} 0.036 \\ 0.009 \\ 0.021 \end{Bmatrix}$$

$$\sigma = \begin{Bmatrix} 480 \\ 120 \\ 280 \end{Bmatrix} N/m^2$$

Solvy

$J_{11} = 1$, $J_{12} = 0$, $J_{21} = 0$, $J_{22} = 0.5$ (7)

$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$|J| = 0.5$$

$$[B] = \frac{1}{|J|} \times \begin{bmatrix} J_{22} & -J_{12} & 0 & 0 \\ 0 & 0 & -J_{21} & J_{11} \\ -J_{21} & J_{11} & J_{22} & -J_{12} \end{bmatrix} \times \frac{1}{4}$$

$$\begin{bmatrix} -(1-\eta) & 0 & -(1-\eta) & 0 & (1+\eta) & 0 & -(1+\eta) & 0 \\ -(1-\varepsilon) & 0 & -(1+\varepsilon) & 0 & (1+\varepsilon) & 0 & (1-\varepsilon) & 0 \\ 0 & -(1-\eta) & 0 & (1-\eta) & 0 & (1+\eta) & 0 & -(1+\eta) \\ 0 & -(1-\varepsilon) & 0 & -(1+\varepsilon) & 0 & (1+\varepsilon) & 0 & (1-\varepsilon) \end{bmatrix}$$

$$[B] = \frac{1}{0.5} \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0.5 & 0 \end{bmatrix} \times \frac{1}{4} \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$= \frac{0.5}{0.5 \times 4} \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & -2 & 0 & 2 & 0 & 2 \\ -2 & -1 & -2 & 1 & 2 & 1 & 2 & -1 \end{bmatrix}$$

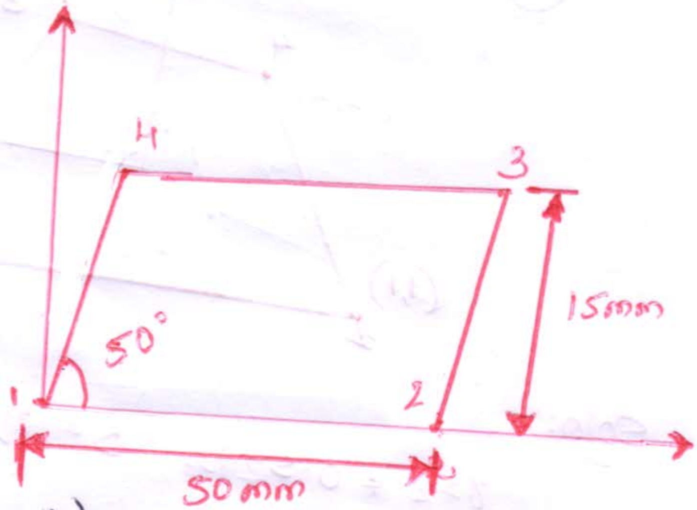
$$[B] = 0.25 \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & -2 & 0 & 2 & 0 & 2 \\ -2 & -1 & -2 & 1 & 2 & 1 & 2 & -1 \end{bmatrix}$$

We know element Stress.

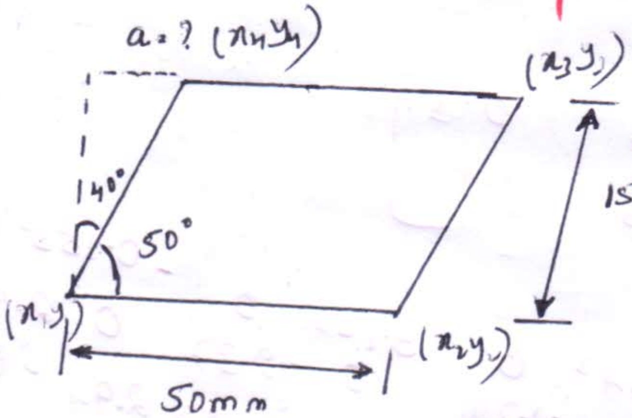
$$\sigma = [D][B]\{u\} \rightarrow (1)$$

(6) Consider a quadrilateral element as shown in figure. The local co-ordinates are $\xi = 0.5$ and $\eta = 0.5$. Evaluate Jacobian matrix and strain displacement matrix.

(HW)



Soln:



$\tan 40° = \frac{a}{15}$

$a = 12.586 mm.$

$(x_1, y_1) = (0, 0)$ $\qquad$ $(x_3\ y_3) = (62.586, 15)$

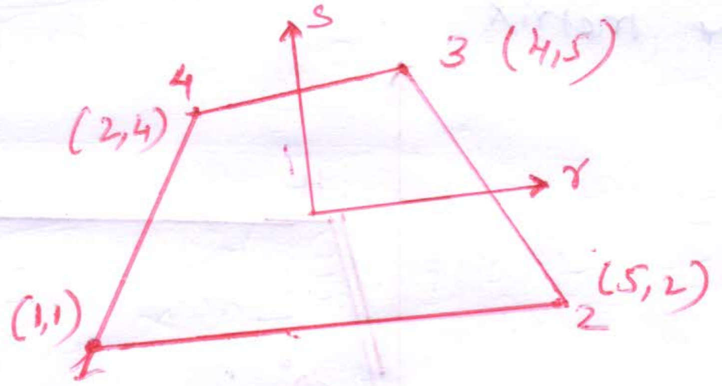
$(x_2, y_2) = (50, 0)$ $\qquad$ $(x_4\ y_4) = (12.586, 15)$

Soln:

(i) $J = \begin{bmatrix} 25 & 0 \\ 6.293 & 7.5 \end{bmatrix}$

(ii) $[B] = \frac{1}{750} \begin{bmatrix} -3.75 & 0 & 3.75 & 0 & 11.25 & 0 & -11.25 & 0 \\ 0 & -9.3535 & 0 & -40.65 & 0 & 28.1 & 0 & 21.94 \\ -9.3535 & -3.75 & -40.6465 & 3.75 & 28.1 & 7.9 & 21.94 & -7.9 \end{bmatrix}$

(7) Establish the Strain-Displacement matrix for the linear quadrilateral element as shown in figure at gauss point $r = 0.57735$ and $s = -0.57735$



Soln:

$r = \xi = 0.57735$, $s = \eta = -0.57735$

$$[J] = \begin{bmatrix} 1.7886 & 0.5 \\ -0.2886 & 1.5 \end{bmatrix}$$

$$[B] = 0.0884 \begin{bmatrix} -2.1547 & 0 & 3.155 & 0 & -0.155 & 0 & -0.845 & 0 \\ 0 & -1.211 & 0 & -2.37 & 0 & 2.9432 & 0 & 0.634 \\ -1.2111 & -2.155 & -2.37 & 3.155 & 2.94 & -0.1457 & 0.634 & -0.845 \end{bmatrix}$$

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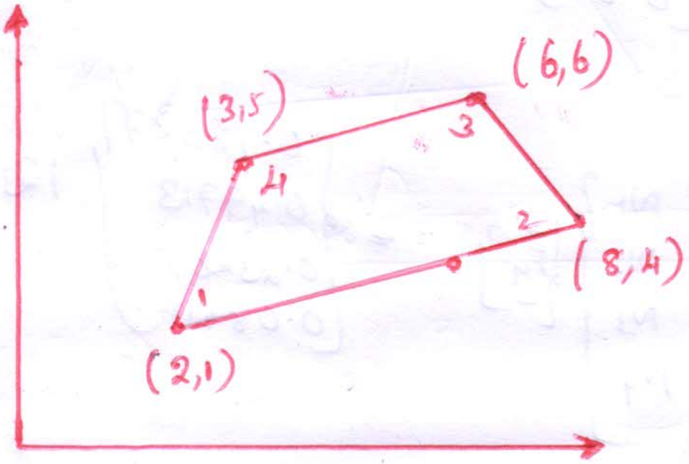
(8) For the isoparametric quadrilateral element shown in figure the Cartesian co-ordinates of point P are (6,4). The loads 10 kN and 12 kN are acting in x and y directions on that point P. Evaluate the nodal equivalent forces.



find
* N_1, N_2, N_3, N_4
* $x =$, $y =$

$F_x = 10kN$, $F_y = 10kN$.

$\eta = -0.362$, $\xi = 0.46$.

$N_1 = 0.18387$, $N_2 = 0.49713$,

$N_3 = 0.23287$, $N_4 = 0.08613$.

Element force vector

$$\{F\}_e = [N]^T \begin{Bmatrix} F_x \\ F_y \end{Bmatrix}$$

$$\begin{Bmatrix} f_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{Bmatrix} \{F_x\}$$

$$= \begin{Bmatrix} 0.18387 \\ 0.49713 \\ 0.23287 \\ 0.08613 \end{Bmatrix} \{10\}$$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{Bmatrix} 1.83 \\ 4.97 \\ 2.33 \\ 0.86 \end{Bmatrix} \text{kN}$$

$$M_1 \quad \begin{Bmatrix} F_{1y} \\ F_{2y} \\ F_{3y} \\ F_{4y} \end{Bmatrix} = \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{Bmatrix} \{F_y\} = \begin{Bmatrix} 0.18387 \\ 0.49713 \\ 0.23287 \\ 0.08612 \end{Bmatrix} 12$$

$$\begin{Bmatrix} F_{1y} \\ F_{2y} \\ F_{3y} \\ F_{4y} \end{Bmatrix} = \begin{Bmatrix} 2.20644 \\ 5.965 \\ 2.794 \\ 1.0335 \end{Bmatrix}$$